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THE INTRODUCTION OF "FORBIDDEN" AMPLITUDES WHEN CALCULATING THE WAVE RESISTANCE OF A SHIP†

E. L. AMROMIN, A. N. LORDKIPANIDZE and YU. S. TIMOSHIN

St Petersburg

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"Forbidden" values of the amplitudes are introduced into the Havelock formula, used in the linear theory of ship waves, which associates the wave resistance with their amplitudes. As a result, it is possible to achieve satisfactory agreement between calcaulation and experiment for various shapes of vessels.

THE WAVE resistance of a vessel depends on the amplitude of the ship waves caused by the vessel. Linear theory assumes that these amplitudes are directly proportional to the intensities of the wave-forming features by which bodies moving close to the free surface are replaced and this enables one to obtain relatively simple formulas for calculating the wave resistance of a ship R_w [1, 2]. However, as the above-mentioned intensities are increased, the experimental dependences for the amplitudes deviate so strongly from linear dependences that the divergence between theory [1, 3] and experiment for R_w turns out to be striking. Attempts to solve the non-linear spatial problem of ship waves both by expanding the flow characteristics in series in powers of a small parameter [4] as well as by using characteristics distributed over its boundaries do not yield satisfactory results in the calculation of R_w for ships of different shapes and different values of the Froude number, Fr, in spite of the considerable computer resources which are used.

In this situation, it is reasonable to appeal to what is probably the simplest method of partially taking non-linearity into account, that is, to the introduction of limiting or "forbidden" amplitudes. Limiting amplitudes, which cannot be exceeded for any intensity of the perturbation source, are encountered in various branches of mechanics and physics.

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In introducing "forbidden" amplitudes it is assumed that waves of any amplitude, including the limiting amplitude, satisfy linear theory; but subsequently the amplitude and, consequently, the energy transferred by the waves reach the limiting values. The theory of free planar surface waves, for example, specifies the amplitude of Stokes waves [2] with a sharp vertex as the limiting amplitude and experiments ([5], for example) agree with this. In this paper, the introduction of forbidden amplitudes and energies into the calculation of the wave resistance of a ship is described and demonstrated using vessels of different shapes as examples.

In the theory of ship waves, the wave resistance of a vessel is expressed in terms of their amplitudes. The elevation of the free surface η in linear theory is given by the formula

$$\eta = Ug^{-1}\partial\varphi/\partial x \tag{1}$$

Here, g is the acceleration due to gravity, U is the velocity of the motion of the vessel along the x-axis and the potential φ caused by the velocity of the fluid is the solution of the boundary-value problem

$$\Delta\varphi = 0,\ U^2\partial^2\varphi/\partial x^2 + g\partial\varphi/\partial z\,|_{z=0} = 0;\quad \partial\varphi/\partial N\,|_D = U\,(N\ x);\quad \lim_{x\to-\infty}|\varphi| = 0 \tag{2}$$

Here, N is the normal to the surface of the vessel D on which a no-penetration condition is satisfied and the z-axis is directed upwards. The analytical solution of problem (2) was obtained by Havelock for a system of characteristics specified by an intensity $Q(x, z)$, distributed on a rectangle in the $y = 0$ plane and the expression for their wave resistance has the form [2]:

$$R_w = \pi\rho U^4 \int\limits_0^{\pi/2} F(\theta)\cos^3 0\, d\theta \tag{3}$$

The function F is the square of the amplitude of the waves, θ is the angle between a point of the wave wake and the diametral plane of the ship and ρ is the density of sater. Formula (3) can be reduced to the form

$$R_w = \frac{\rho\nu^2}{\pi}\int\limits_0^{\pi/2} |H(k,\theta)|^2 \frac{d\theta}{\cos^3\theta},\quad H(k,\theta) = \int\limits_{-L/2}^{L/2}\int\limits_{-T}^{0} Q(x,z)\,\theta^{kz+ikx\cos\theta}\,dx\,dz \tag{4}$$

where H is the Kochin function, $\nu + gU^{-2}$, L, B and T are the length, width and draft of the ship and $k = \nu\cos^{-2}\theta$. As is usual in linear theory, we assume that

$$-Q = 2U\,(N,\ x) \tag{5}$$

If relation (5) is further simplified, neglecting the square of the derivatives of the ordinates of the hull f, then

$$Q = 2U\partial f/\partial x \tag{6}$$

and (4) reduces to the extensively used Mitchell formula [1, 3], the generally accepted dimensionless version of which has the form

$$c_w \equiv \frac{2R_w}{\rho U^2 S} = \frac{2Fr^2L^2}{\pi S}\int\limits_0^{\infty} \sigma(u)\,du,\quad \sigma(\theta) = \frac{|H(k,\theta)|^2}{U^2L^4\cos\theta\,\mathrm{Fr}^8} \tag{7}$$

Here S is the area of the wetted surface of the ship, $u = \mathrm{tg}\,\theta$, $\mathrm{Fr} = (L\nu)^{-1/2}$ and the function σ determines the relative fraction of the energy which is transferred by linear waves behind the ship. The calculation of c_w using formula (7) for a single value of Fr using the method in [6] takes a few seconds on a personal computer. The form of the function σ in the case of a Wigley vessel with an L/B ratio of 10 and an overall fullness (with a ratio of the water displacement to the LBT product) $\delta = 0.44$ is shown in Fig. 1 for two different cases: when

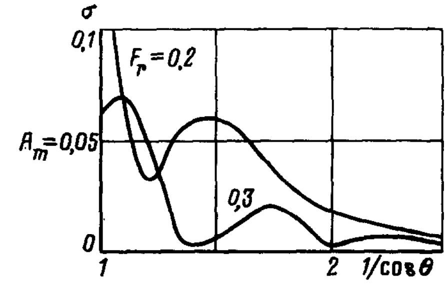

FIG. 1.

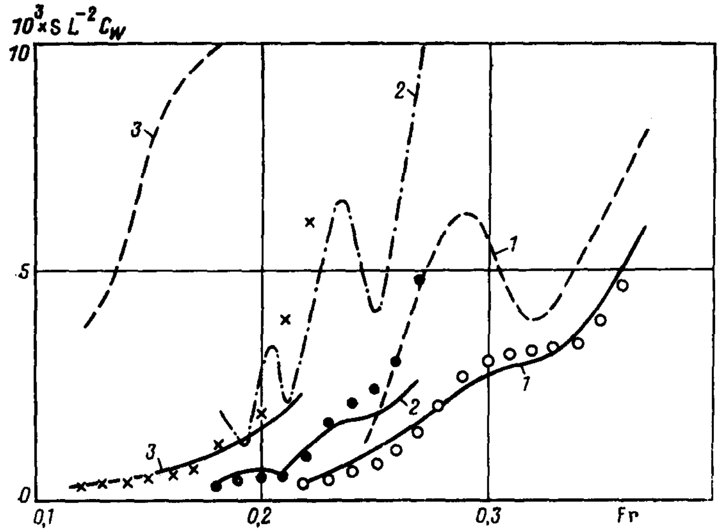

FIG. 2.

Fr = 0.2, energy is predominantly transferred by transverse waves in the wake of the ship, while when Fr = 0.3 their interference is favourable and energy is predominantly transferred by diverging waves which correspond to values of θ close to the Kelvin angle.

The unsatisfactory agreement between calculations using formulas (6) and (7) and experiment was pointed out a long time ago in [1]. Figures 2 and 3 illustrate the present position of work involving calculations of c_w. The experimental dependences of the wave resistance coefficient c_w on Fr for three models of the 60th series [3] which are the most frequently used as standards are shown in Fig. 2: for $\delta = 0.6$ and $L/B = 7.5$ (the open circles), $\delta = 0.7$ and $L/B = 7$ (solid circles) and for $\delta = 0.8$ and $L/B = 6.5$ (the small crosses). For all models, $B/T = 2.5$. The results of calculations using the basic formula of linear theory, that is, Mitchell's formula, are shown by the dashed lines. The experimental points depicted by open circles in Fig. 3 refer to the Wigley model [3] with analytical contours while the experimental points depicted by the solid circles refer to a modern container carrier with $L/B = 4.8$, $\delta = 0.55$ and $B/T = 3.5$. The broken lines in this figure are for a calculation using formulas (6) and (7) and the values of c_w which are given by such a calculation for this container carrier are so large that they exceed 0.002 even when Fr = 0.15.

The range of Froude numbers, which are considered in the examples and are typical of commercial ships, is distinguished by the fact that more than a single wavelength λ of the ship waves is now piled up within the limits

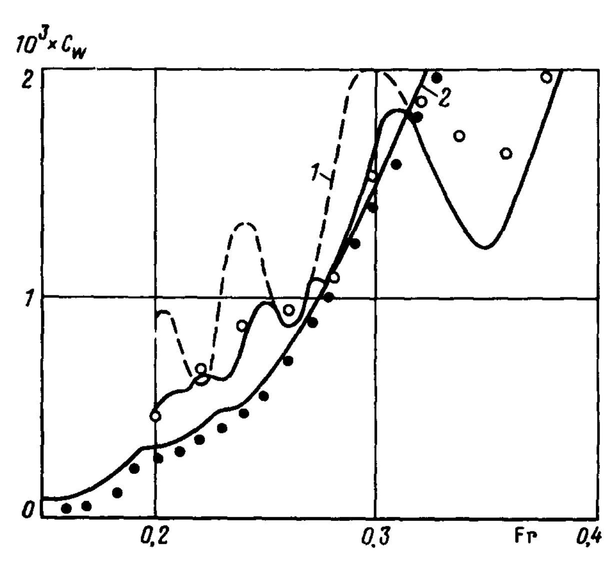

FIG. 3.

of the extension of the hull. Wave interference effects therefore turn out to be very substantial. The linear theory, as the results of an exact numerical solution of elementary planar problems [7] show, can appreciably distort these effects, mainly by significantly increasing (by a factor of two) the amplitudes of the waves even for fixed intensities of the characteristics.

In non-linear theory, at present it is only sufficiently reliable to determine just the phases of the waves while the values of c_w are determined with large errors by means of lengthy calculations. This is apparently because, in direct numerical methods, the joining of the local non-linear solutions with the asymptotic solutions for the distant wave wake has still not been realized on account of the fact that the law of conservation of energy is apparently not satisfied. The small-parameter method [4] also led to completely paradoxical results (see the dashed curve in Fig. 2).

The facts which have been mentioned above suggest the idea of maintaining the structure of formula (6) while so correcting the function σ in it so as to take account of the non-linearity of the interference of the ship waves. The simplest way of making this correction, which does not necessitate the direct solution of non-linear problems in potential theory, is to introduce "forbidden" amplitudes into (6) or, more accurately, their limiting ratios to λ. It is well known [2] that this ratio depends slightly on λ, that is, on Fr. It is therefore natural to introduce constraints precisely on the relative energy $\sigma(\theta)$. Instead of (7), it is necessary to use the function

$$\sigma(\theta) = \min\left\{\frac{|(Hk,\theta)|^2}{\mathrm{Fr}^8 L^4 U^2 \cos\theta}, A_m\right\} \tag{8}$$

where the number A_m must be independent both of the shape of the ship and of the Froude number. The correction (8) of the function (6) is shown in Fig. 1 by the dashed line. Not having the means to determine A_m theoretically, we chose it on the basis of a numerical experiment involving a single example and checked it on many others. The results of the checks, the solid lines in Figs 2 and 3, were obtained using formulas (4), (6) and (8) for one and the same value $A_m = 0.05$. The results of linear theory for a contemporary bulk carrier with $\delta = 0.74$, $B/T = 3.8$ and $L/B = 4.3$ are in such poor agreement with experiment that it is difficult to compare them in the graph: for values of Fr = 0.13, 0.14, 0.15, 0.16, 0.17, 0.18, 0.19 and 0.20, for experimental values of the quantity $10^3 SL^{-2} c_w$ of 0.04, 0.05, 0.06, 0.09, 0.15, 0.23, 0.34 and 0.52, linear theory gave the values 4.51, 4.57, 5.30, 5.24, 6.66, 6.91, 7.50 and 9.43 while the calculation with "forbidden" amplitudes gave 0.035, 0.050, 0.074, 0.091, 0.121, 0.165, 0.210 and 0.252.

While there is good agreement between the calculated and experimental values of c_w, which is ensured by the modification proposed here of the formulae of linear theory, there is, of course, still not complete agreement. Complete agreement would not be expected: in fact, the non-lineaity of the initial problem has been taken account of in terms of just a single interdiction (8). The present differences in c_w are explicable. For instance, the fact that the calculated values of c_w for a Wigley vessel are low when Fr = 0.3 is associated with the fact that a transverse bow wave occurs at Fr = 0.3 with an opposite phase to the stern wave and, in such cases, as is well known from planar theory [7], linear interference reduces R_w. The pronounced increase in the experimental relations c_w (Fr) when Fr is increased for vessels with $\delta \geqslant 0.7$, beyond which the solid lines in Fig. 2 do not keep up, is associated with the fact that, in traditional experimental methods for determining R_w using the towing resistance, the models do not distinguish between what is strictly the wave resistance (the amounts of energy consumed in the elastic oscillations of water) and the wavebreaking resistance, which is not described within the framework of potential theory, the friction of which is greater, the higher the value of Fr and the blunter the waterline of the vessel. We note that, from a practical point of view, the agreement between the calculation using formula (8) and experiment is sufficient since, on account of wavebreaking, vessels are not used in practice when $\mathrm{Fr} > 0.6 - \delta/2$ ([3], p. 397). Meanwhile, even in these interesting practical cases, the introduction of "forbidden" amplitudes cannot provide an answer to the question regarding the shape of the streamlines and completely replace non-linear theory although, in these cases, it is already possible to speak about the controlling role of formula (8) which is similar, let us say, to the role of the Squire–Young formula in the theory of separated flows [8].

In concluding, we note that what has been described here is not the first attempt to achieve an improvement in the agreement between linear theory and experiment by means of a constraint of a Kochin function as Inoui had already laid the foundations of this [3]. However, he did not succeed in revealing the correct substantiation of such corrections. For example, it is difficult to analyse a recent attempt [9] to explain the divergence between linear theory and experiment by the failure to take account of the effect of viscosity since, in [9], the narrowing of the displacement body in the near wake which reduces its cross-sectional areas by a factor of three to four has been ignored.

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THE PERTURBATION METHOD IN A SPATIAL PROBLEM OF THE LINEAR VISCOELASTICITY OF ANISOTROPIC BODIES†

T. S. KAGADII, L. V. MASSAKOVSKAYA and A. V. PAVLENKO

Dnepropetrovsk

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A generalization of an asymptotic method, which has been developed earlier [1, 2] as applied to elastic materials, to the case of viscoelastic anisotropic media is proposed. The problem of the transmission of a load to viscoelastic orthotropic bodies by elastic elements, which is associated with the adhesive strength of composite fibre materials, is investigated.

1. CONSIDER a viscoelastic body consisting of a material which is orthotropic both with respect to its elastic and its viscoelastic properties. The principal directions of anisotropy coincide with the Cartesian axes of the x, y, and z coordinates. In this case, the relationships between the strains and the stresses can be written in the following manner:

$$
\begin{gathered}
e_{11} = S_1 - \nu_{12}S_2 - \nu_{13}S_3 \\
S_i = \frac{1}{E_i}\left(\sigma_{ii} + \int_0^t K_{ii}(t-\tau)\,\sigma_{ii}\,d\tau\right), \quad i = 1, 2, 3 \\
e_{ij} = \frac{1}{G_{ij}}\left(\sigma_{ij} + \int_0^t K_n(t-\tau)\,\sigma_{ij}\,d\tau\right)
\end{gathered}
\tag{1.1}
$$

$(i = 2,\ j = 3,\ n = 1;\ i = 1,\ j = 3,\ n = 2;\ i = 1,\ j = 2,\ n = 3)$

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